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ABSTRACT

An appeal for more research to determine how to educate children as effectively as possible is made. Mathematics teachers can readily examine the educational problems of today in their classrooms since learning progress in mathematics can easily be measured and evaluated. Since mathematics teachers have learned to think in quantitative terms and make decisions based on logical thinking, they can provide the best quantitative thinking about our educational problems. There is a need for people who think quantitatively and objectively and apply their thinking to meet the needs of all people. Examples of problems related to such topics as space exploration, environmental pollution, and the cost of living are used to illustrate how mathematics teachers can explore these problems with their students in a quantitative and logical manner. (FL)

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AT THE
GOLDEN JUBILEE YEAR ANNIVERSARY MEETING
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
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SHERATON-PARK HOTEL
WASHINGTON, D. C.

QUANTITATIVE THINKING

I take pleasure in greeting this great group of teachers of mathematics, gathered together -- as teachers have gathered together from time immemorial -- to ask each other how they can be better teachers. You are eagerly seeking to hear of the experience of others as to how they have helped our young people in schools across the country to learn more, to understand more about your particular subject. And you are anxious to share what you have learned.

You are all aware of the fact -- else you would not be here -- that nurturing the minds of the younger generation is one of the most important and one of the most difficult tasks which human beings face. How little we know really about how young minds develop, how children learn, what makes them want to learn, why some learn some things faster than others -- but other things more slowly. We all know that under some teachers, students learn rapidly and enthusiastically -- while other teachers with apparently similar children find only apathy and slow learning.

We are all aware of -- but only dimly understand -- the great differences in how different children learn. Some learn most quickly by doing, others by listening, others by seeing or reading, still others by some mysterious combination. How can each child's special tastes and skills be brought out, his interest excited? How can we take into account the fact that some children have learned a lot -- and have come to like to learn -- even before they enter school Others have had no such advantage. Can this latter handicap ever be overcome? If so, how?

Or, is it true that learning habits are established before the age of 5 and somehow we must find better ways to help the very young child?

These are unanswered questions -- and to a discouraging extent we do not even know how to go about finding the answers. Yet, find them we must. The educated young man or woman is far too important to our society to trust to luck that we are now educating all our children as effectively as possible.

In his recent message on education President Nixon strongly urged that we give far greater attention to the research necessary to find answers to these and similar questions.

In the frontier days of our country the little red school house filled our needs pretty well -- possibly because we did not expect too much of it -- and partly because our society then really didn't need an efficient educational system, as much as it does now.

Times have changed. We left the little red school house long ago, and we now have -- in some communities at least -- magnificent modern buildings with facilities once undreamed of. But do we really have any quantitative information as to how much these fine facilities have added to the effectiveness of the learning process? Or to the enthusiasm with which the children learn? Maybe we need much more extensive and expensive facilities -- but maybe we don't! There are today something like 51 million students in elementary and secondary schools, nearly 8 million in colleges and universities, and several million pre-school children who are <u>not</u> learning. We spend over $60 billion a year on the educational enterprise -- over $1,000 per year per student on the average. How important it would be if we could get the same quality of education at 20% less cost -- or even better, at the same cost, to get a 20% better education. That's a 12 billion dollar question!

It is in the field of mathematics that many of these problems come to a focus. And mathematics teachers are therefore in a key position to examine objectively and help solve these problems. Why is this?

First, it is in mathematics that the great differences in learning ability and motivation show up most clearly.

Second, it is in mathematics that learning progress can be most readily measured and evaluated.

Third, it is among mathematics teachers that we should find the best quantitative thinking about educational problems.

Thus, every mathematics class room can be a small research laboratory in which one looks for individual differences, identifies them, seeks to discover the source of these differences. Could some students who seem to learn slowly be helped to learn faster with mechanical aids -- blocks, dominoes, games, simple adding machines, or visual aids? Could the youngster who insists he doesn't like math be made to like it by similar aids or new approaches? For example, young boys worship the astronauts; can they be interested in some of the problems the astronauts face: how fast are they going? how long till they get there? how long will the food last? And at advanced levels there are hosts of problems about acceleration, weightlessness, navigation, the decreasing pull of the earth, the increasing pull of the moon, etc., etc. I think we all know that the first requisite to learning is a desire to learn, an interest in learning.

Math teachers in a given school can conduct different experiments in different classes -- and compare results. The same experiment could be conducted in schools in different parts of town representing different pre-school experiences. And, of course, at national meetings such as this you can compare experiences across the country -- and make sure that everyone knows the results -- so all can use them.

Now, I know that a large number of you do just these things all the time. But you have no idea how important it is that you do them -- and do them even more extensively. Not only to attain better learning in mathematics, but to illuminate the mysteries of the whole learning process -- for the benefit of all teachers and all students everywhere.

I am prejudiced, of course, but I believe the teachers of mathematics -- in collaboration with their colleagues in science -- can make a great impact on the progress of education in this country. You know what quantitative thinking is, what quantitativ asurements mean, how controlled experiments are made and evaluated and analyzed. You know about errors and uncertainties and how to evaluate or minimize them. You know how <u>not</u> to draw unwarranted conclusions from too little or too uncertain data.

You are, in short, able to think in quantitative terms and to guide your conclusions and your actions through logical thinking.

And how our country needs today -- as never before -- people who can think quantitatively and objectively! And what a responsibility rests upon you as mathematics teachers to help all students recognize the value of quantitative thought -- and be able to do a little bit of it!

How many of the puzzles and problems that we face, and worry about and quarrel about, could be resolved if only people were able to stop and add 2 and 2 together and find the right answer!

Let me give a few examples. I won't bother to discuss the more obvious everyday problems we all face in financial transactions; e.g. which is cheaper 6 1/2 ounces at 17¢ or 9 ounces at 22¢? Let me use examples in the field of environmental pollution -- which everyone is talking about these days.

A space expert recently gave a speech in which he mentioned the cost of lifting a pound of weight into earth orbit -- about $500. And he said that with some new technologies coming along this would be reduced by a factor of 10 -- to $50 a pound. A questioner -- obviously worried about pollution -- asked: When it's <u>that</u> cheap, why can't we shoot all our waste material into the sun and let it burn up there? The space expert was patient. He said you could. To go to the sun would cost $5,000 per pound, $10 million per ton. But the city of Washington produces 3,000 tons of waste per <u>day</u>, and the whole country produces 3 billion tons per <u>year</u>. Cost per year: 30 million <u>billion</u> dollars. That's 30,000 times the gross national produce! Our cities have a hard time digging up $2 a <u>ton</u> to take care of trash.

Another environmentalist heard with horror that a couple of hundred tons of DDT was being washed into the oceans every year. "All the life in our oceans will be destroyed," he insisted. It is not difficult to estimate that the total weight of all the ocean water is such that 200 tons of DDT would "contaminate" the ocean to 1 millionth of a part per billion (10^{-15}). No living thing could possibly even detect, much less be harmed, by that concentration, even after 100 years.

Now, I am not in favor of wholesale dumping of DDT into the ocean. But let's not get hysterical about imaginary dangers.

I was recently mentioning that it should be possible to reduce the contamination of our air and water by a factor of 10 in the next 30 years. I was told rather rudely that this would be no good -- because 100 million more people would wipe out the gain. But 100 million people is only 50 % more than we have at present. How can that wipe out a factor of 10 improvement? I guess my questioner could not compare the number 10 with the population number of 100 million. One hundred million is bigger than 10, isn't it? Yes -- if you can't really think in quantitative terms and do not understand the meaning of the ratio of two numbers.

The laws of exponential growth are also a continuing puzzle to many people. Almost anything that grows seems to grow exponentially -- population, gross national product, the cost of living, pollution, the balance in a savings account (if there are no withdrawals), the consumption of electric power - etc., etc.

It all sounds excessively simple. My savings account earns compound interest at 5 %. Does that mean if I deposit \$100 that after a year I have \$105, after ten years \$150, after 100 years \$600 (i.e. \$100 + \$500)? Not at all. After 10 years I have \$165, after 100 years not \$600 but \$14,841, and after 1,000 years I have or my descendants will have nearly $\$10^{22}$, i.e. ten billion times the present GNP.

The lesson from this is, of course, that exponential growth cannot continue at the same rate forever. If your savings account earns 5 % and the gross national product grows at 4 %, then some day your account will exceed the GNP. If the population grows at 2 % per year and the food supply grows at 1 %, then some day starvation will be rampant -- and population growth will be slowed down or stopped. Thus nature -- or human ingenuity -- finds some mechanism of limiting any exponential growth rate. Interest rates are lowered, birth rates decline, or death rates go up -- something intervenes before catastrophe sets in. It would be instructive for your students to explore these questions.

I read recently that a group of college students, to show their interest in reducing air pollution, raised $2500 to purchase a new car -- and then buried it in the ground so it could not pollute the air. If the students had been a bit more quantitative-minded, they would have learned that a new 1970 car produces 5 times less pollution than did a new 1965 car and 10 times less than a used 1965 car. But a used 1965 car could have been purchased for $500, let us say. Therefore, if they had used their $2500 to purchase and bury 5 used 1965 cars, they would have reduced air pollution by 50 times as much as by burying a 1970 model.

It would have been still better to have buried 10 used 1960 cars. Wouldn't that have made more sense?

Furthermore, if they had taken these 5 cars to a scrap steel dealer, they could have recovered 5 tons of steel for eventual re-use -- instead of throwing away and wasting one ton.

But let us move to still deeper questions.

What is it that makes modern American civilization different from the days of the cave man, the days of the middle ages, or even of 200 years ago? What indeed makes America of today different from India, China, Central Africa or many other places?

The difference is simply that Western man, about 200 years ago, began to understand nature's laws and learn how to use them. He began to understand that the stars and planets were not pushed capriciously across the sky by gods or angels or devils or gremlins, but that they obeyed a fixed and understandable set of rules. The planets were governed by the same laws of motion which governed a rolling ball or a falling apple on the earth.

And most of all, men learned that these forces and motions could be related <u>quantitatively</u>. It was not just a "pretty" idea that Newton had that the motion of the moon and of a falling stone has an aesthetically pleasing relation to each other. They did indeed have that. But the reason for the aesthetic beauty was that the <u>numbers</u> came out right. The equations relating the changing motion (that is, the acceleration) of the moon were the same equations which described the motion of a falling body. And when you measured quantitatively the numbers in one case, and inserted them in the equations, you got the right numbers for the other case. Nature thus became predictable -- not in a general philosophical sense as the early Greek philosophers believed, but in an exact numerical sense. The motion of the moon or the planets was not "something like" that of a falling stone. They were identical. They fit the same equations -- not just nearly but <u>exactly</u> -- within the errors of observation. Our understanding of nature need no longer be expressed in vague philosophical presumptions; it could be expressed in relations between <u>numbers</u>. Thus, one could now design a machine and know in advance how it would behave. Later scientists learned not only the laws of mechanics but also of optics, of electricity, of thermodynamics, of the behavior of atoms and molecules -- again in terms of numbers. Science tells us how much, how many, how big, how fast, how far, how soon. It does not simply say a baseball is "quite a bit" bigger than an atom. It says that a baseball is as big as 10^{27} atoms. It does not say if you burn a ton of coal you can get "quite a lot" of electrical energy. It tells us that under specified conditions a ton of coal can produce a specific number of kilowatt-hours of electrical energy. Science doesn't say simply that if you push hard enough on a rocket you might get to the moon. It tells us precisely how much fuel to use and how to arrange things to get to a particular spot on the moon -- and can specify the time of arrival within a second.

Now, whether we know it or not, nearly everything we do in our daily lives depends on the fact that some one has figured out the numbers involved. Driving a car, heating our house, placing a long distance call, turning on the lights, riding an airplane, building a house or a school or a skyscraper -- someone has used the known laws of science and engineering to make it work -- and figure the cost. Whether we figure out these things on the back of an envelope or by using a $5 million computer makes little difference -- except for the time it takes. We must first understand the principles we use and the mathematical ways in which they can be applied to specific problems.

I have spoken above as though the laws of science and engineering were exact and could be expressed in terms of exact numbers. To a substantial degree this is true. In a very large number of cases we can predict the behavior of a mechanical or an electrical system to an astounding degree of accuracy. I have already indicated that we can measure the velocity of a spacecraft and compute the gravitational forces on it sufficiently accurately as to determine within a second when it will reach a particular spot on the moon and we can predict that spot on the moon to possibly 100 feet precision. When we consider that the spacecraft has traveled 240,000 miles that's pretty good predicting. We could do even better if the fallible human beings that push the buttons could time the pushing more precisely. There are many physical laws which predict events so accurately that no matter how precisely we make the measurements we find no deviation.

However, as we dig deeper we recognize that no measurement of any physical quantity can ever be made with absolute accuracy -- that is to the point where we can be sure that the error is exactly zero. If we use a meter stick to measure the width of a desk top, it is no trouble to measure the length to, say, a sixteenth of an inch. If the desk is 36 inches long, that means an accuracy of about one-sixth of one per cent. That's plenty good enough for most purposes. If we wanted to do better we could use a more accurate yard stick, possibly employing a magnifying glass or a vernier and we might be able to measure the length to 1/100th of an inch. Here, however, we run into difficulties. If we made our measurement on a different portion of the desk top we would probably find that the length varied slightly from one side of the desk to the other. Or again if we had measured the length on two different days when the temperature was different we might find that there had been a slight contraction or expansion. Nevertheless, we can measure lengths under suitable conditions to an astonishing degree of accuracy. If we had a rigid non-expandable metal bar carefully polished on the two ends, we could, using optical methods, measure the length to within 1/100,000th of an inch or better -- an accuracy, say, of one part in ten million. Recent experiments have been made with laser beams reflected from the special reflector left on the moon by the Apollo 11 astronauts and these measurements can determine the distance to the moon, 246,000 miles, within a few inches.

Nevertheless, no matter how hard we try we always come to a limit of the accuracy of any particular measurement; therefore, if we want to test the equation which represents the physical law by measuring the various quantities in the equation we will always find that there is residual uncertainty in our measurements and, therefore, a residual uncertainty in the accuracy of the law we are testing. Maybe the uncertainty is one per cent, maybe a thousandth of a per cent, maybe it is only one part in a hundred million.

The physical scientists estimate their errors of measurement with very great care and they always express their findings in such a way as to indicate as carefully as they can the degree of uncertainty in their conclusions.

This has had most important consequences in certain cases. The motion of the planet Mercury around the sun was found many years ago to be not quite in agreement with the numbers calculated from Newton's laws of motion and his law of gravitation. At first the discrepancy was attributed to error or measurement. As measurement techniques improved, however, and became more accurate the discrepancy became more and more certain. It was not until Einstein's theory of relativity came along that the discrepancy was explained and a slightly modified form of Newton's law was thereby established. Measurements of the motion of

Mercury now agree with this law, but who can say that even tinier discrepancies may not some day be found forcing a still further correction.

Now the point of this little excursion is simply to emphasize the fact that any number which is based upon physical observations has within it a certain degree of uncertainty, depending upon the accuracy of our measuring technique. The careful scientist however determines these uncertainties with great care and is meticulous in expressing just what the uncertainties are.

He knows too that for some purposes uncertainties, of, say, 1 % are relatively unimportant and he does not go to great trouble and expense to make his measurements more carefully than required for his purpose.

Now there are certain kinds of numbers that we bandy around all the time without regard to the uncertainties inherent in them. When the results of the census, taken on April 1st, are eventually published, we will probably be told that the number of people in the United States was, say, 203,463,527. No one really believes that the last numbers are 527 rather than

528 or 550 or, indeed, some considerably larger or smaller number, for on April 1st many babies were born and many individuals died. Furthermore, we are sure that the census takers will have missed some and possibly counted others twice.

For most purposes small errors would make no difference -- possibly even an error of one per cent, that is two million people out of 200 million would not be important. I would not be surprised, indeed, if the error might be about that large in spite of the census takers insistence on expressing the number to six significant figures, suggesting that his measurement was good to one part in 200 million which is obviously impossible.

Probably the errors in counting the number of people in the country are less than the errors inherent in many other kinds of social and economic statistics which we are using all the time. How accurate can we really measure the gross national product? The cost of living index? The number of tons of steel produced in the United States per year? The amount of money which American tourists spend in Paris every year? Or the number of Americans which subsist on less than, say, 1500 calories of food intake per day?

For most purposes we don't need to know such numbers with high precision. And yet every once in a while we find that people are using such numbers as though they were accurate to one per cent or a tenth of a per cent, or even better. What does it mean, for example, when someone says that the cost of living advanced .3 of a per cent last month and only .2 of a per cent this month? How are those numbers measured and do they really represent a change in your cost of living or mine?

Social and economic numbers have another drawback. They are not only subject to often unknown errors but there are no exact laws which relate the values of these quantities one month to the value to be expected the next month or the next year. We can make an intelligent guess as to how much the cost of living may go up this year, but as everyone knows the expert can be badly fooled. He has no Newton's laws of motion on the basis of which he can make accurate and dependable predictions. Many a business has failed, many a political policy has gone wrong because predictions of the future were taken at their face value and not regarded as educated prognostications.

In fact the difference between social science and physical science is precisely that in the one case there are laws by which one can predict the behavior of the physical system with high numerical precision and in the other case the laws are unknown or have a high degree of numerical uncertainty. The lesson from this is simply that numbers are extremely valuable but one must examine them carefully to determine what degree of confidence he can have in their precision.

Nevertheless, we are fortunate that there is a great body of science and technology for which accurately verified laws are available and on the bases of these laws plus the skilled use of mathematics we have built a great and flourishing technological civilization in the Western World.

Now some people claim they don't like our modern technology-based civilization. And there are some unhappy features about it -- such as air and water pollution. But I doubt if any of us would really trade places with a cave man -- or a peasant, or even a prince, of the middle ages. Technology has done many things to make life better. But we must learn to use technology still better. That we can do -- providing more people who understand technology put their minds to its more considerate use. And provided more citizens, more Congressmen, more businessmen and more teachers recognize that science is the quantitative understanding of nature and technology is the quantitative application of science to meet quantitatively the needs of all people.